LETTERS TO THE EDITOR.

**** Correspondents are requested to be as brief as possible. The writer's name is in all cases required as proof of good faith.*

'A singular optical phenomenon.'

THE 'singular optical phenomenon' described by 'F. J. S.' on p. 275 of the current volume of *Science* is a case of the familiar *watering* effect produced by superposed loose and regular fabrics, or by distant palings and lattice-works superposed by projection. We may find it convenient, in the following discussion, to refer to these by the general term of 'projection phenomena,' although the phrase does not seem to me to have much to recommend it except convenience.

I ought to say that this discussion is prompted by the letter by Professor LeConte in the last number of *Science;* for, if so skilled an experimenter could overlook the real explanation, it may safely be concluded that most readers have done so. Moreover, the phenomenon is one of a large and interesting class, of which I have never met any explanation, although, as we shall see, very simple considerations will lead us far towards a complete explanation of all.

For the sake of simplicity, we will begin by the consideration of two gratings of regular horizontal elements: the one nearer the observer, which we will call the first grating, is to be of alternating opaque and transparent strips; and the more distant one, or second grating, of white and black bands. We will also suppose, at first, that the eye is placed in a line passing through the middle of a dark band and an opaque strip, and that the aperture of the pupil is negligibly small. We may also conveniently assume that the angular widths of the elements of both gratings are so small that they are not separately evident to the eye, not only because such cases offer the most striking phenomena, but also because in them the meaning of the term 'apparent brightness,' which we shall use, is self-evident.

We will call the distances from the eye to the screens respectively d_1 and d_2; the breadths of the opaque and black intervals, b_1 and b_2; and, finally, the element of each grating (that is, the distance from the centre of one dark strip to the centre of the next), E_1 and E_2.

If B is the brightness of the white portion of the second grating, it is evident that the average brightness of the field, if the first grating were removed, would be

$$B\frac{E_2-b_2}{E_2}.$$

If, on the other hand, the first screen remained in place, and the black strips of the second should be replaced by white of brightness B, the field would appear of a brightness

$$B\frac{E_1-b_1}{E_1}.$$

As a first special case, let us suppose

$$\frac{E_1}{d_1}=\frac{E_2}{d_2}:$$

then, remembering the position of the eye, it is clear that each opaque bar would be centrally projected upon a dark strip of the second grating; and the brightness would be uniform, and equal to the less of the two expressions above.

For a second case, suppose

$$\frac{E_1}{d_1}=n\frac{E_2}{d_2},$$

n being any whole number: then every nth black strip would be centrally covered by a bar of the first grating. If $\frac{b_1}{d_1}$ is equal to or less than $\frac{b_2}{d_2}$, the brightness would be uniform, and equal to $B\frac{E_2-b_2}{E_2}$; but, if this limit of equality were surpassed, the average brightness would be

$$B\frac{nE_2-(n-1)\,b_2-b_1\frac{d_2}{d_1}}{nE_2},$$

and there would be regularly placed minima, unless the angle $\frac{nE_2}{d_2}$ were insensible to the eye.

The case of $n\frac{E_1}{d_1}=\frac{E_2}{d^2}$ is equally easy.

In all that follows, we will, in order to avoid too extensive discussion, regard n as equal to unity: by this limitation we sacrifice no interesting cases.

Suppose, now, the eye moved continuously up or down, parallel to the gratings. After a certain small displacement, depending upon the relation of $\frac{b_1}{d_1}$ to $\frac{b_2}{d_2}$, the brightness of the field would continuously diminish until it reached a minimum equal to

$$B\frac{E_2-b_2-b_1\frac{d_2}{d_1}}{E_2},$$

unless the numerator should be negative, when the minimum would be absolute. It would remain at this minimum for a certain time, depending upon the constants of the system, and then increase by exactly the same law as that of decrease, until after a displacement of the eye equal to $E_1\frac{d_2}{d_2-d_1}$, when it would recur to the same condition as at first.

As a final and more general case, let us suppose that

$$\frac{E_1}{d_1}=\frac{E_2+\delta}{d_2},$$

where δ is a small quantity, positive or negative. If we again suppose that the eye is so placed that a line drawn from it perpendicularly to the two gratings will pass centrally through dark bars in each, then a line drawn from the eye through the mth bar of the first grating will pass through a dark strip of the second, if $\frac{m\delta}{d_2}$ is a whole number. Let m be the *smallest* number which meets this condition: then a line drawn through any bar between the 1st and mth would meet some one of the conditions discussed in the last paragraph, as produced by a movement of the eye. Thus we see that the field would present horizontal maxima and minima of brightness, the angular position (θ) of the maxima being given by the equation

$$\theta=\operatorname{tang}^{-1}N\frac{mE_1}{d_1}$$

where N is any whole number, positive or negative. The apparent distance apart of the maxima would be $\frac{mE_1}{d_1}$.

If the eye be moved so as to shift the apparent position of the central bar to the adjacent black strip on the second grating, the middle of the field would have undergone all the changes of phase which correspond to a change of tang θ from zero to $\frac{mE_1}{d_1}$; hence such a motion of the eye would appear to give

rise to a shifting of the whole series of maxima by this angle. The direction of apparent motion would be either with that of the eye, or opposite, according as δ is positive or negative. The displacement of the pupil necessary to bring about this change would be

$$E_2 \frac{d_1}{d_2 - d_1}.$$

If the relative motion of the periodic phenomenon and the first screen be regarded as a parallactic displacement, then we must suppose their relative distances from the eye inversely proportional to their apparent motions; i.e., as

$$\frac{mE_1}{d_1} \text{ to } \frac{E_2}{d_2 - d_1};$$

or, since $\frac{E_1}{d_1} = \frac{E_2}{d_2}$ nearly, as m to $\frac{d_2}{d_2 - d_1}$.

It was this apparent parallax which led 'F. J. S.' to suppose the phenomenon which he describes an image of the distant screen between himself and the first window.

If our gratings be complicated by the addition of equally spaced vertical bars, we shall see also, in general, a series of vertical bands giving maxima and minima along a horizontal direction. These will be separated by intervals

$$\frac{m' E_1'}{d_1};$$

and the ratio of their apparent angular motion to that of the first screen when the eye is moved equals

$$m' \text{ to } \frac{d_2}{d_2 - d_1},$$

where the letters marked with $'$ are defined by analogy.

A very interesting conclusion follows from the consideration that m and m' are wholly independent; the one depending on δ, and the other on δ'. Thus, we may have the horizontal bands moving in the same direction as the eye, and the vertical bands moving in the opposite direction, or *vice versa:* hence, if the displacement of the eye is neither horizontal nor vertical, the network which forms the projection phenomenon may seem to move in any direction, the only condition being that the horizontal and vertical components of the velocity are proportional, respectively, to m' and m; or, in other words, to the apparent width of the bands, divided by the corresponding element of the first grating.

In the case of gratings which are not plane, superposed by projection, as is the condition generally with doubled laces, veils, mosquito-bars, etc., — in short, in almost all cases of every-day observation, — both δ and δ', as well as the direction of the elements of the gratings, are functions of the distances from the central point of the field; but, as these are continuous functions, we can state several of the most important properties of the projection phenomena: viz., —

1°. The bands will be continuous and curved. 2°. If the eye be moved, the phenomenon will shift with an apparent velocity in any direction proportional to the width of the bands measured in that direction. 3°. The motion of a single band will, in general, be a motion of translation, combined with a motion of rotation. But the instantaneous centre of rotation cannot lie in a band; for in that case, according to the previous conclusion, that point being at rest, the band would there have no width, consequently could not exist. 4°. If a band forms a closed curve, a motion of the eye will necessarily produce a continuous change in the apparent magnitude of the ring; for a mere motion of translation would correspond to a momentary rotation about at least two points in the curve, which, according to the last principle, is impossible.

The properties described under the second and fourth heads above are those which more especially cause the projection phenomena to resemble those of watered silk; for the latter follow much the same law.

We will now consider the effect of the size of the pupil of the observing eye, which has hitherto been considered as a point. It is obvious that the image on the retina must be the sum of the projection images as seen from each point of the pupil: hence, if the pupil is not much greater than the space through which the point of view must be shifted in order to produce a complete change of phase (i.e., than $E_2 \frac{d_1}{d_2 - d_1}$), the phenomenon must be like that for an indefinitely small pupil, except that the discontinuity is less pronounced. This explains why, in fine networks, such as veils and mosquito-bars, the distance $d_2 - d_1$ between the fabrics must be small in order to produce the projection phenomena. In the case described by 'F. J. S.,' $E_2 = \frac{1}{2}$ inch, $d_1 = 10$ feet, and $d_2 = 40$ feet: consequently the expression indicating the limit which the diameter of the pupil must not greatly surpass is $\frac{1}{6}$ inch.

The effect of maladjustment of the eye would be to diminish still further the discontinuity of the phenomenon; but this would be carried so far as to destroy the periodicity, and thus obliterate the phenomenon, — not when an angular interval of $\frac{E_2}{d_2}$ at the distance d_2 becomes indistinguishable, as 'F. J. S.' seems to have expected, but only when an angular interval of $\frac{mE_1}{d_1}$ at the distance d_1 becomes indistinguishable.

The cases where n differs from unity offer no difficulties, but they are much less interesting. They exclude the case which has given rise to this discussion; for there E_1 equals $\frac{1}{8}$ inch, the other dimensions having been already quoted.

In what precedes, however, I have tacitly assumed that $\frac{\delta}{d_2}$ is always the reciprocal of a whole number. This may not be true. Suppose the value to lie between $\frac{1}{N}$ and $\frac{1}{N+1}$, where N is a whole number: then, if N is large, the solution above is accurate within the range of observation. If, on the contrary, the value of N is moderate, successive maxima will differ by a quantity which is itself periodic.

It will be observed that the second grating may be perfectly replaced by an image by reflection of the first. Frequent examples of this arrangement are seen in screens before closed windows or mirrors.

The general analytical solution of the whole class of phenomena produced by parallel rectangular gratings with indefinitely small pupil is easy; but the solution is so extremely general, that its reduction to special interesting cases requires even more writing than we have found necessary here. The only point worth dwelling upon here is, that the apparent variations in brightness, though periodic, are always discontinuous; but that every departure from the assumed geometrical conditions, such as are effected by diffraction, dimension of the pupil, and imperfect accommodation, tends to decrease the discontinuity.

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Baltimore, April 11.